Key to Tutorial 6
Problems of Combinational Logic

Exercise 1 Hot Drink Vending Machines

A hot drink vending machine provides coffee or tea, with or without milk, or milk alone. Three buttons control the machine: "coffee," "tea," and "milk." To get a single drink, simply press the corresponding button. To get a drink with milk, press both the button for the desired drink and the "milk" button simultaneously. Furthermore, the machine only works if a token has been inserted into the slot. Pressing no button or an incorrect action after inserting the token (for example, pressing "coffee" and "tea" simultaneously) will result in the token being returned and no drink being dispensed. Since milk is free, the token is also returned if milk alone is selected.

Let A, B, C, and D be the input logical variables corresponding to the following propositions:

- A = 1 iff the "coffee" button is pressed
- B = 1 iff the "tea" button is pressed
- C = 1 iff the "milk" button is pressed
- D = 1 iff a token has been inserted into the machine's slot

Let W, X, Y, and Z be the logical output variables corresponding to the following propositions:

- W = 1 iff coffee is dispensed
- X = 1 iff tea is dispensed
- Y = 1 iff milk is dispensed
- Z = 1 iff the token is returned

Note that the return of the token can be active or inactive when no token is inserted into the machine (you should use the Φ symbol).

1. Write down the truth table for this system.

Button			Token	Dispensing			Returning the token
Coffee	Tea	Milk		Coffee	Tea	Milk	
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	Φ
0	0	0	1	0	0	0	1
0	0	1	0	0	0	0	Φ
0	0	1	1	0	0	1	1
0	1	0	0	0	0	0	Φ
0	1	0	1	0	1	0	0
0	1	1	0	0	0	0	Φ
0	1	1	1	0	1	1	0
1	0	0	0	0	0	0	Φ
1	0	0	1	1	0	0	0
1	0	1	0	0	0	0	Φ
1	0	1	1	1	0	1	0
1	1	0	0	0	0	0	Φ
1	1	0	1	0	0	0	1
1	1	1	0	0	0	0	Φ
1	1	1	1	0	0	0	1

2. Using Karnaugh maps, give the simplest expressions for the outputs.

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	1	1	0

W = A.$\overline{B}$.D

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

X = $\overline{A}$.B.D

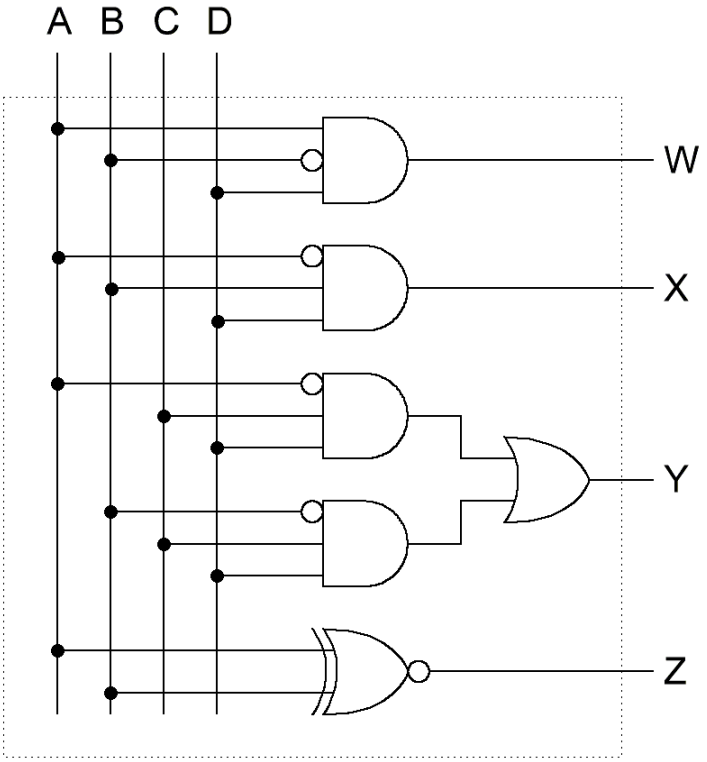
AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	0	0
10	0	0	1	0

Y = $\overline{A}$.C.D + $\overline{B}$.C.D

AB \ CD	00	01	11	10
00	Φ	1	1	Φ
01	Φ	0	0	Φ
11	Φ	1	1	Φ
10	Φ	0	0	Φ

Z = $\overline{A}.\overline{B}$ + A.B = $\overline{A \oplus B}$

3. Draw the circuit diagram.



Exercise 2 Liquid Level

Let us consider two tanks: *T1* and *T2*. The liquid level of each tank is checked by two sensors: a high-level sensor (*A* for *T1*, *B* for *T2*) and a low-level sensor (*C* for *T1*, *D* for *T2*). The values of *A*, *B*, *C*, *D* are 1s when there is some liquid in front of the sensor; otherwise they are 0s. Three indicator lights (*V1*, *V2*, *V3*) are set according to the following conditions:

- *V1* = 1, if *T1* and *T2* are full.
- *V2* = 1, if *T1* and *T2* are empty.
- *V3* = 1, in any other cases.

1. Write down the truth table for this system. *A*, *B*, *C* and *D* are the inputs. *V1*, *V2* and *V3* are the outputs. If some cases are not possible, use the Φ symbol.

A	B	C	D	V1	V2	V3	
0	0	0	0	0	1	0	
0	0	0	1	0	0	1	
0	0	1	0	0	0	1	
0	0	1	1	0	0	1	
0	1	0	0	Φ	Φ	Φ	← Not possible
0	1	0	1	0	0	1	
0	1	1	0	Φ	Φ	Φ	← Not possible
0	1	1	1	0	0	1	
1	0	0	0	Φ	Φ	Φ	← Not possible
1	0	0	1	Φ	Φ	Φ	← Not possible
1	0	1	0	0	0	1	
1	0	1	1	0	0	1	
1	1	0	0	Φ	Φ	Φ	← Not possible
1	1	0	1	Φ	Φ	Φ	← Not possible
1	1	1	0	Φ	Φ	Φ	← Not possible
1	1	1	1	1	0	0	

In this truth table, some conditions never occur. If a high-level sensor is 1, the low-level sensor of the same tank cannot be 0. In other words, if *A* is 1, *C* cannot be 0; and if *B* is 1, *D* cannot be 0.

2. Using Karnaugh maps, write down the most simplified expressions of *V1*, *V2* and *V3*.

V1 (AB \ CD)	00	01	11	10
00	0	0	0	0
01	Φ	0	0	Φ
11	Φ	Φ	1	Φ
10	Φ	Φ	0	0

V1 = A.B

V2 (AB \ CD)	00	01	11	10
00	1	0	0	0
01	Φ	0	0	Φ
11	Φ	Φ	0	Φ
10	Φ	Φ	0	0

V2 = $\overline{C}.\overline{D}$

V3 (AB \ CD)	00	01	11	10
00	0	1	1	1
01	Φ	1	1	Φ
11	Φ	Φ	0	Φ
10	Φ	Φ	1	1

V3 = $\overline{A}.D + \overline{B}.C$

3. Draw the circuit diagram.

